on calculation. Thus, $r = 1 \times 10^{-8}$, which is the generally accepted order of magnitude of the radius of a hydrogen atom.

In connection with our use of the period of the line H_α in this paper, it is interesting to note that on Ritz' theory of the origin of spectral series, this line is given by the vibration of an electron under the influence of a suitably placed elementary magnet which, according to Ritz, is not inconsistent with the magneton itself.

An Outline of a Theory of Magnetic Storms.

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§ 1. This paper briefly outlines some results of a study of magnetic storms, the object of which was to ascertain their broad general features, leaving aside individual details and cases. The account is limited in several respects, mathematical developments, in particular, being omitted; this is because the paper is intended to be followed by a more complete discussion, when conditions permit. In this future memoir I hope to deal more adequately with many points here lightly passed over, and especially with disturbance phenomena in polar regions, where magnetic conditions are scarcely ever quiet, and where the divergence between the magnetic and geographical axes of the earth introduces most complication.

Part I.—*The Magnetic Data.*

§ 2. Apparently all great world-wide magnetic storms commence simultaneously to within a few seconds,* over the whole earth, although small local magnetic fluctuations may sometimes mask the commencement at particular stations. It is therefore possible, where we are not concerned with very small time intervals, to speak of "storm time," measured from the beginning of a storm, without reference to any individual locality. In contradistinction to storm time, the same for all stations, is local time, which we shall reckon from local midnight.

§ 3. The principal average features of great magnetic storms, with which

* *Cf.* 'Proc. Phys. Soc. Lond.,' May, 1918, and the references there cited.

any theory of these phenomena must first concern itself, appear to be as follows:—

(*a*) Over the whole earth, except, perhaps in the close vicinity of the poles, the net mean change in the horizontal force, during the first half-hour or so, is an increase. This is succeeded by a decrease of much greater amplitude, which lasts for several hours. A period of recovery then follows, and lasts for several days. Both the decrease and the recovery proceed most rapidly shortly after their initial stages, and gradually slow down.

The vertical force is slightly increased in numerical magnitude, both north and south of the equator, during the first day of a storm. Owing to the small amplitude of the change, it is more difficult than in the case of the horizontal force to determine the periods of increase and of subsequent recovery. The declination, or direction of the horizontal force, is hardly affected systematically (relative to storm time) in low and middle latitudes. In high latitudes a small change is observable, comparable in force units with that shown by the vertical force. These changes will be designated the general storm variations. They are approximately independent of longitude or local time, their course being governed only by storm time.

(*b*) The general storm variations are superposed on diurnal variations characteristic of each station. These depend on local time, but differ from the variations observed on ordinary days in a very definite manner. The residual diurnal variations at any station, obtained after subtracting the ordinary local-time changes corresponding to the place and season from the total diurnal variations occurring during a storm, will be termed the local storm variations. These are most intense during the first day of a storm, and gradually subside. There are also indications of a partial suppression of the ordinary diurnal variations. In the main, however, the local storm variations, in each element and at all stations, consist of purely diurnal sine waves, and the phase relations of these waves in the different elements are notably simple.

In the horizontal force the wave attains its maximum at about 6 h. local time, both north and south of the equator, up to a high latitude,* where it vanishes and changes sign.

The vertical force attains its (numerical) maximum at 18 h. approximately in all latitudes, north and south of the equator. The amplitude vanishes in the neighbourhood of the magnetic equator, where, of course, the sign of the

* In this paper, where latitude, pole, or axis is mentioned, the reference is to magnetic rather than geographical latitude, etc. By the magnetic axis is meant the axis of the main spherical harmonic component of the earth's field, and not the line joining the poles of magnetic dip.

variation, considered radially, is reversed. The maximum amplitude, which is greater in this than in either of the other two elements, is attained in high latitudes, where the horizontal force wave suffers reversal.

The wave in declination, or westerly force, similarly changes sign at the equator, its maximum in the northern hemisphere occurring at approximately 12 h. local time. Its amplitude is greatest in high latitudes.

(*c*) The course of the above regular (general and local) storm variations is complicated, and partly masked, by irregular fluctuations in the magnetic force. These will be referred to as the irregular storm variations. They are least frequent and intense at low latitudes, increasing greatly as the poles are approached. The more intense the storm, however, the lower is the latitude to which the irregular storm variations extend. This feature, it will be recalled, is paralleled in the case of auroral displays, which, in temperate latitudes, are so closely associated with magnetic storms.

The irregular variations do not cease at the epoch after which recovery begins in the general storm variation (*a*); they may persist for a day or more beyond this point.*

§ 4. The remarks of the last section are illustrated by figs. 1–4, which summarise the results of a study of the general and local storm variations at 12 observatories during 40 magnetic storms of considerable, though not outstanding, intensity. Not all the observatories, however, afforded data for the total number of storms. The storms occurred during the period 1902–1911, and were selected from those with sudden commencements, tabulated by Mr. E. W. Maunder from the Greenwich records,† and by the Coast and Geodetic Survey from the records at Honolulu, Cheltenham (U.S.A.), and Sitka (Alaska).

Particulars of the 12 observatories, and also of Bombay (§ 7), are given in the following Table. The observatories are arranged in order of latitude, reckoned from the point 80° N., 70° W., as pole. This is the approximate position of the pole of the first spherical harmonic component of the earth's

* In connection with the above general survey of the course of a magnetic storm, reference may be made to the discussion of storms at Bombay by Dr. N. A. F. Moos ('Colaba Magnetic Data, 1846–1905,' Part II, Ch. X, 1910). Dr. Moos confined himself almost entirely to the study of the Bombay data, and did not recognise the nature of the local storm variation in declination and vertical force, but his discussion has been of the greatest service in the present research, and the method of treatment of the data here adopted is almost identical with that first used by Dr. Moos. While I had independently decided upon similar lines of treatment before reading his work, the real stimulus to their application came from the discovery, made about the same time, of the success which had attended their use by Dr. Moos. It is remarkable how few storms it is necessary to investigate in order to ascertain the general and local storm variations.

† E. W. Maunder, 'M. N. R. Ast. Soc.,' vol. 65 (1904).

magnetic field (*i.e.*, the pole of that part of the field which corresponds to a uniform magnetisation); it is also the approximate position of the "auroral pole," or mean centre of the isochasms, according to Fritz.*

Name.	Lat.	Long.	Mag. lat.	H.F.	V.F.	Dec.	Dip.
	°	h.	°			°	°
Batavia	6 S.	7·1 E.	16 S.	0·37	0·22	1 E.	31 S.
Porto Rico	18 N.	4·4 W.	28 N.	0·29	0·34	2 W.	49 N.
Bombay	19 N.	4·9 E.	11 N.	0·37	0·16	1 E.	23 N.
Honolulu	21 N.	10·6 W.	21 N.	0·29	0·24	9 E.	40 N.
Zikawei	31 N.	8·1 E.	21 N.	0·33	0·34	3 W.	46 N.
San Fernando	36 N.	0·4 W.	41 N.	0·25	0·35	16 W.	54 N.
Cheltenham	39 N.	5·1 W.	49 N.	0·20	0·56	5 W.	70 N.
Baldwin	39 N.	6·3 W.	48 N.	0·22	0·56	9 E.	69 N.
Pola	45 N.	0·9 W.	45 N.	0·22	0·39	9 W.	60 N.
Greenwich	51 N.	0·0	55 N.	0·19	0·43	16 W.	67 N.
Potsdam	52 N.	0·9 W.	54 N.	0·19	0·43	9 W.	66 N.
Sitka	57 N.	9·0 W.	61 N.	0·16	0·57	30 E.	75 N.
Pavlovsk	60 N.	2·0 E.	58 N.	0·17	0·47	1 E.	71 N.

§ 5. Fig. 1 illustrates the course of the general storm variations in all elements, the curves being derived by tabulating the hourly values during the first two days of each of the 40 storms according to the nearest hour of storm time. The results are given for three groups of observatories, in low, middle, and high latitudes.†

The force and time scales are sufficiently indicated in the figure, but it should be mentioned that the storm time corresponding to any dot on the curves is really half-an-hour less than that scaled at the foot. The first point on the curve is obtained from the hourly values of the element for the hour which, in each case, next precedes the commencement of the storm; hence, in the mean, it refers to $(-0\frac{1}{2})$ h. storm time—and similarly for the remaining points, which follow consecutively at hourly intervals. In figs. 3–5 on the contrary, the time scales are strictly correct.

The curves give the hourly means of the differences of the hourly values of the magnetic elements during a storm from the monthly mean values for the months including the storms. In horizontal force, it will be noticed, the first value, half-an-hour before the storm, is somewhat above the mean

* *Cf.* 'Encyc. Brit.,' vol. 2, p. 928; the isochasms are the curves of equal frequency of occurrence of auroræ.

† It would perhaps have been better to interchange Porto Rico and Zikawei in these groups, which were originally chosen according to geographical latitude. The results for Sitka have not been included in the high-latitude group, owing to the influence of irregular disturbance on the general storm variation curves deduced for this station; at Sitka the local storm variation much exceeds the general storm variation, especially in vertical force.

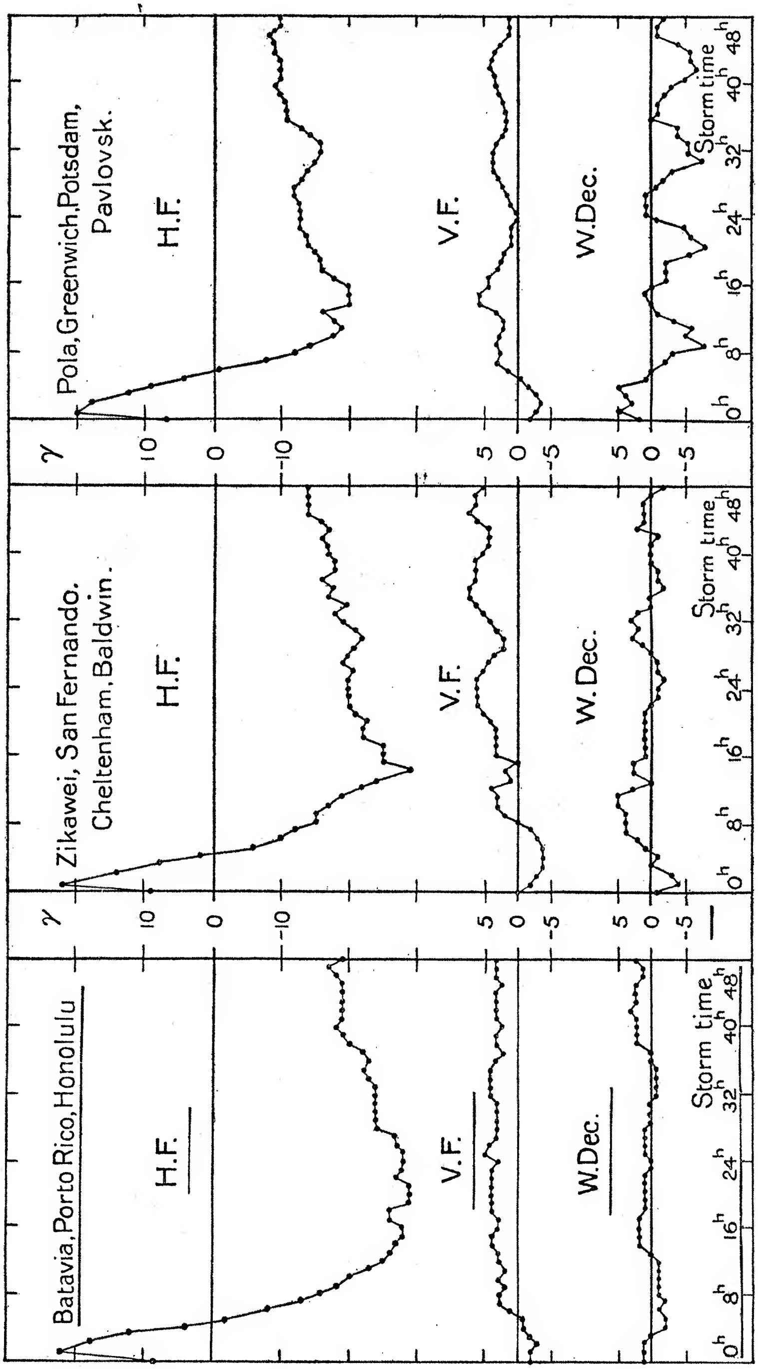

FIG. 1.

monthly mean (which in all cases in fig. 1 is represented by the horizontal line forming the zero on the force scale); this is due to the depression of the

monthly mean by the continued defect of force after the storm, and enables an estimate to be made of the duration of the period of recovery. The first value represents the ordinary normal better than the mean monthly mean for these particular months.

The storm variation in horizontal force does not show very much change with latitude up to about 50°, beyond which point the change becomes much more marked (*cf.* § 7). In the moderate storms here dealt with, at the end of the first half-hour the force is about 14γ ($1\gamma = 0{\cdot}00001$ C.G.S.) above its initial value. A decrease of from 50γ (at the tropics) to 40γ (in mean European latitudes) then follows, over a period of about 20 hours at tropical stations, where, owing to the greater smoothness of the curves, the time is most readily determinable. From the results described in § 7, the duration of decrease appears to be greater, and the amount of the decrease decidedly less, in high latitudes. The recovery after the decrease has ceased lasts for many days, only about 10γ having been recovered by the end of the second day in the cases depicted in fig. 1. This recovery shows itself in the non-cyclic variation on quiet days, though it begins, and proceeds most rapidly, at a time when the irregular storm variations may still be vigorously active.

The vertical force curves show a small increase in force during the first day of a storm. The subsequent slow recovery accounts for the non-cyclic variation in this element on quiet days.

The declination curves show no appreciable general storm variation, except at high latitudes, where, mixed up with a great deal of accidental irregularity, there appears to be a systematic diminution of westerly force. It is uncertain how far this is true for all stations in high latitudes.

The horizontal force changes seem to apply strictly to this element, and are not a component of changes really relating to (geographical) north force; if the latter were the case, where the declination amounts to several degrees the component of the north force changes which should appear in the declination would be appreciable. In order to test this point, the declination variations at Honolulu, San Fernando, Cheltenham, Baldwin, Pola, Greenwich, and Sitka, have been resolved along the geographical

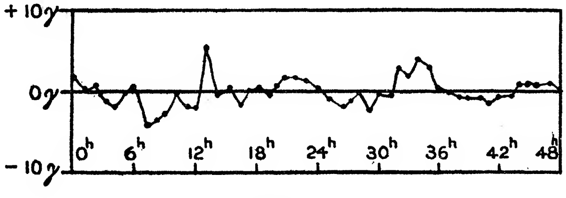

Fig. 2.

meridian and added together, the sum being divided by the sum of the sines of the declinations. The result is shown in fig. 2, which should agree with the horizontal force curves in fig. 1 if the above hypothesis were correct. This is clearly not the case.

It should be added that, in figs. 1, 3, 4, and 5, the vertical force and the declination at Batavia (south of the equator) have been measured positive upwards and to the east respectively, contrary to the convention adopted for northern stations.

§ 6. The curves in figs. 3, 4, 5, show (*a*) the average ordinary monthly mean diurnal variations in the three elements for the months of the 40 storms considered, and (*b*), (*c*) the additional mean diurnal variations (the local storm variations) during the first and second days respectively of the storms. The latter (*b*, *c*) are partly included in the former (*a*), since these variations (*a*) are derived from all days, including those of storm. Thus, if more normal "ordinary" diurnal variations had been adopted as datum curves, the local storm variations would have been slightly larger; the difference is unimportant for our present purpose, however, which is merely descriptive, though the inclusion of all days makes a serious difference to the ordinary diurnal variation at Sitka and Pavlovsk.

Neglecting slight irregularities, the local storm variation curves have each a single maximum and minimum, occurring approximately at the times mentioned in § 3. The purely diurnal character of the curves would probably appear still more clearly if only a fraction (80 or 90 per cent.) of the ordinary diurnal variations had been abstracted from the total diurnal variations during the storm [the latter are, of course, obtainable at once by the addition of the variations (*a*) to (*b*) or (*c*).]

The five sets of curves in figs. 3, 4, 5, refer to the following stations:—

(1) Sitka, magnetic latitude 61°.
(2) Pavlovsk, magnetic latitude 58°.
(3) Mean of Pola, Potsdam, Greenwich—mean magnetic latitude 51°.
(4) Mean of Zikawei, San Fernando, Cheltenham, Baldwin—mean magnetic latitude 40°.
(5) Mean of Batavia, Porto Rico, and Honolulu—mean magnetic latitude 22°.

The curves for Sitka and Pavlovsk have been smoothed by taking means of consecutive overlapping groups of three hourly values.

It will be seen that in type and distribution the ordinary and the additional "storm" diurnal variations are quite different. The most remarkable feature of the latter is the large amplitude of the variation in

vertical force at high latitudes, which far exceeds anything observed in the other elements, for the stations dealt with. In the ordinary diurnal

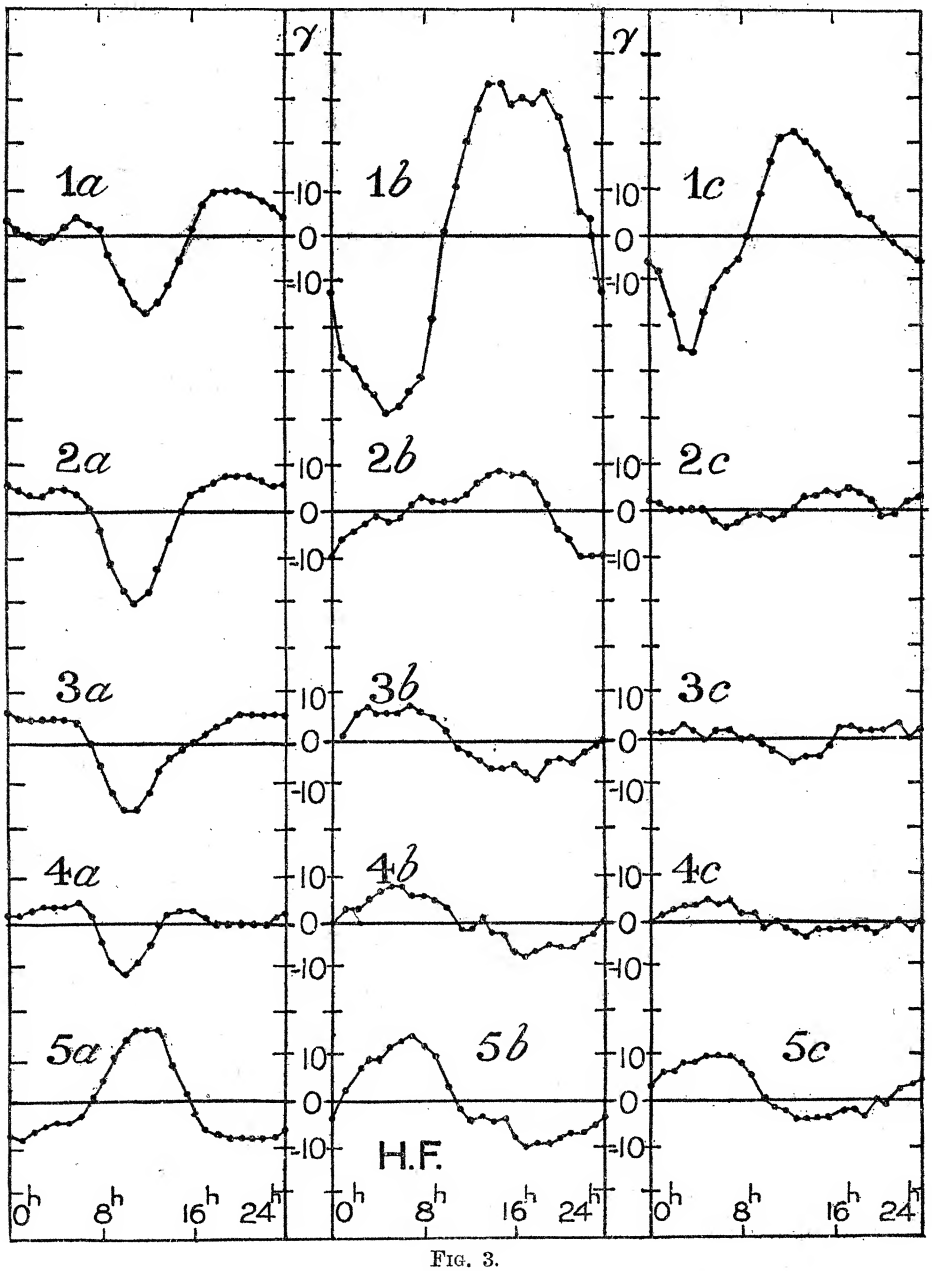

Fig. 3.

variations, of course, the vertical force changes are markedly less than the force variations in the horizontal plane. The rapid increase in the

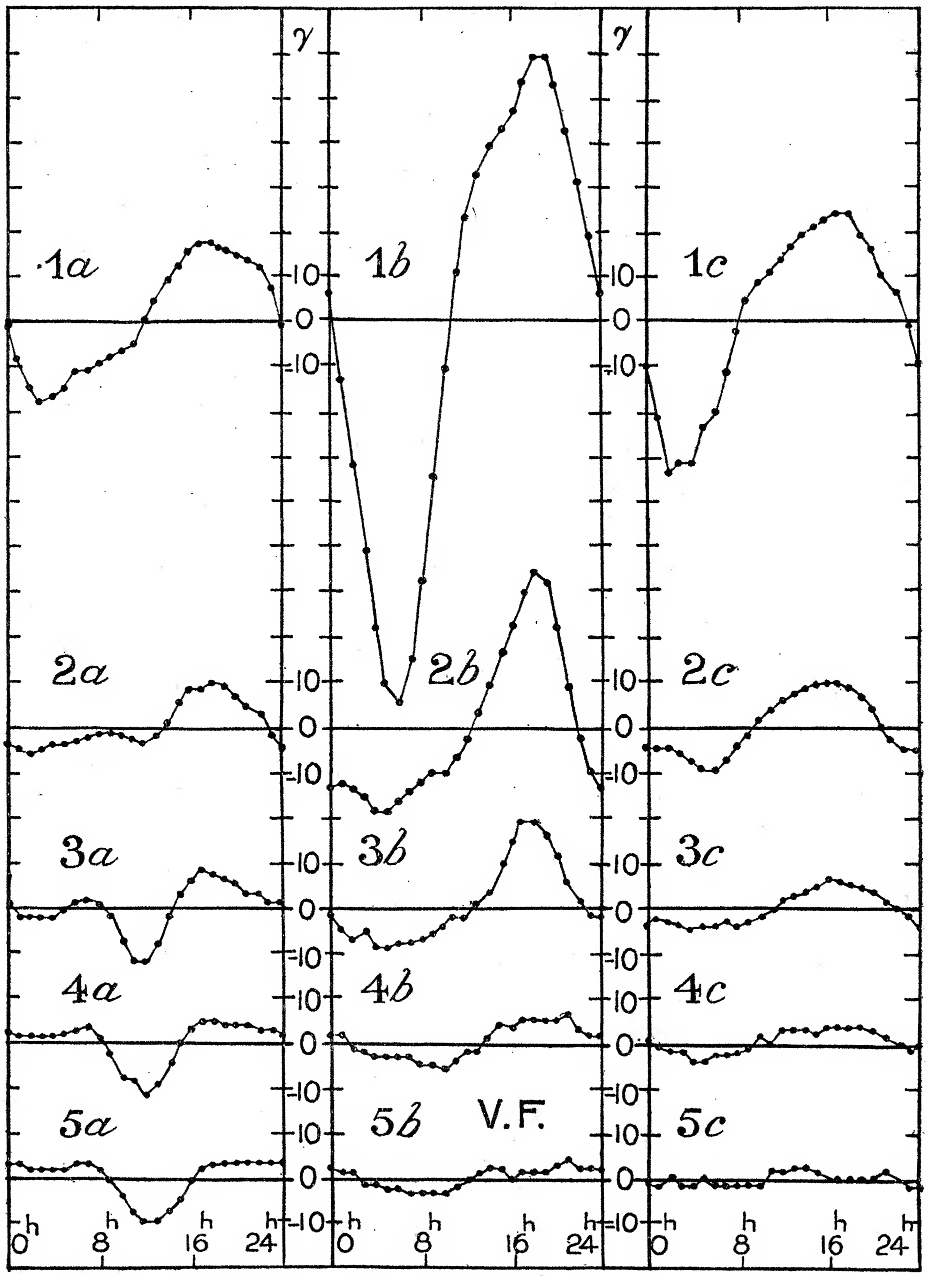

FIG. 4.

horizontal force variation (*b*) after its reversal between latitudes 50° and 60° is also noteworthy. In all cases the local storm variations show a considerable diminution from the first to the second day.

§ 7. The influence of intensity on the character of the general and local storm variations is illustrated by fig. 6, which shows the variations—in

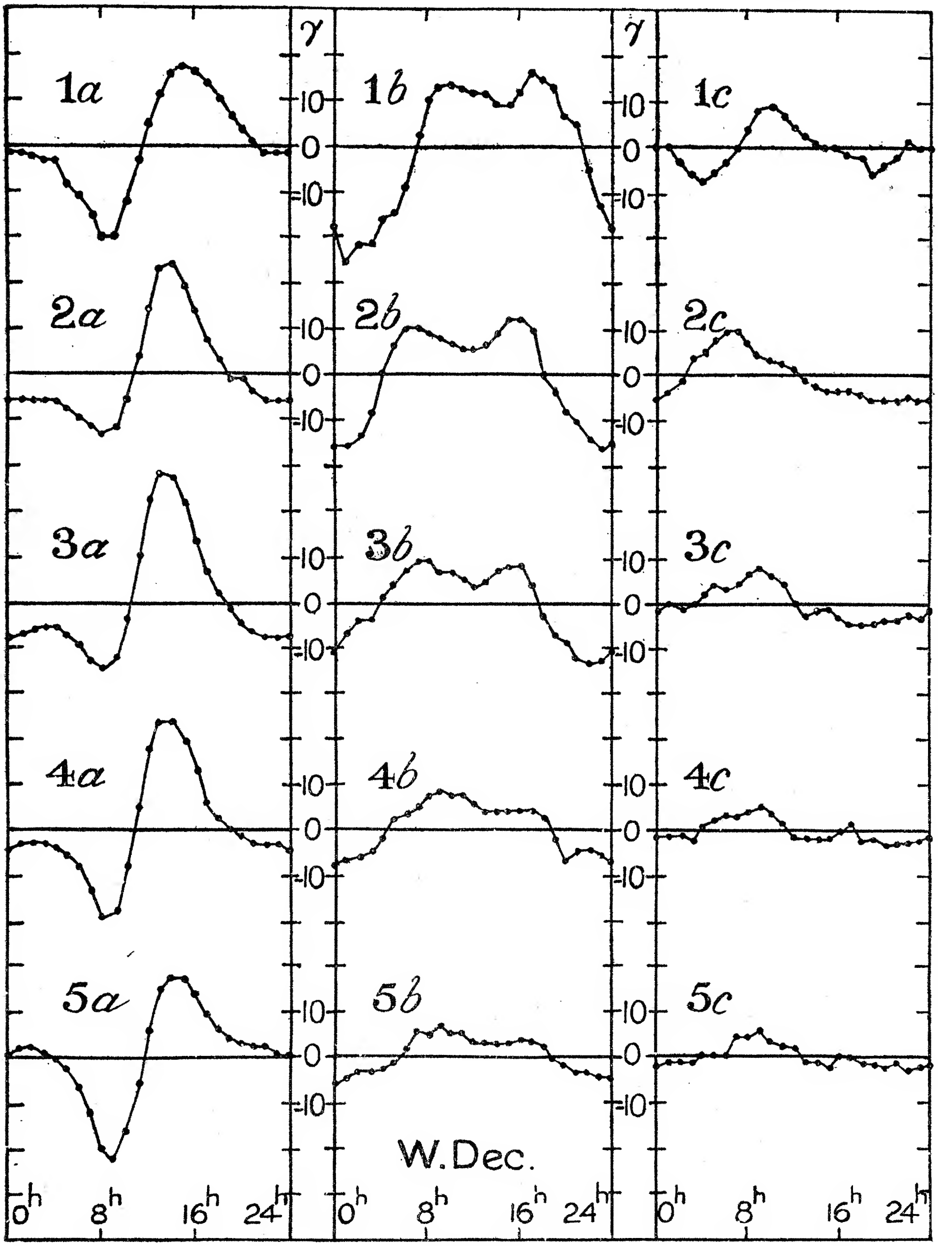

Fig. 5.

horizontal force only—for three groups of storms of widely differing intensities. The storms, 110 in number, are among those already partly

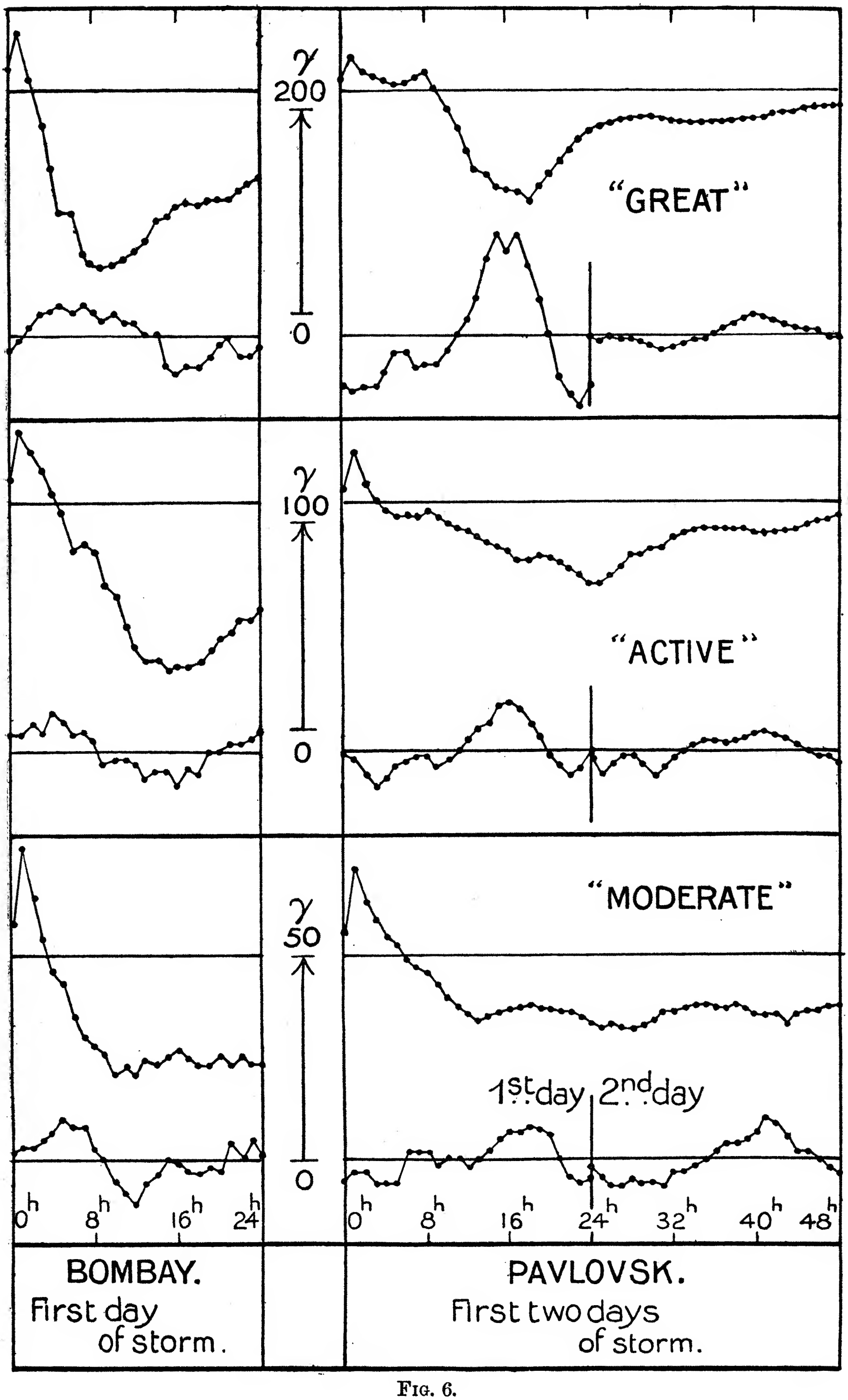
γ
200
0
"GREAT"
γ
100
0
"ACTIVE"
γ
50
0
"MODERATE"
1st day
2nd day
0h
8h
16h
24h
0h
8h
16h
24h
32h
40h
48h
BOMBAY.
First day
of storm.
PAVLOVSK.
First two days
of storm.

Fig. 6.

discussed by Dr. Moos (*loc. cit.*); they are drawn from the period between 1872 and 1904. The Bombay horizontal force data for the first day of the storms, given in the volume already referred to, have been used, and, in addition, the corresponding Pavlovsk observations (for the first two days of the storms, however). A few of the earlier storms were not covered by the Pavlovsk data, the total number treated in the latter case being 95. The storms were grouped according to the total horizontal force range at Bombay on the first day, there being 31 "great" storms, with a range of over 225 γ, 33 "active" storms, with a range between 225 γ and 150 γ, and 46 "moderate" storms, with a range less than 150 γ. The general storm variation (relative to storm time, as in figs. 1, 2) and the local storm variation (relative to local time, as in figs. 3, 4, 5) for each group, and for the two stations, are shown in fig. 6, but the force scales used in the three cases are in the ratios 1 : 2 : 4, as indicated, in order to reduce the curves to similar dimensions throughout. The Bombay curves are unsmoothed, while the Pavlovsk general and local storm curves are smoothed by taking means of five and three consecutive values respectively.

The most noteworthy features of the diagram are (*a*) the increasing lateness of the epoch of minimum horizontal force, with diminishing storm intensity and also with increasing latitude; (*b*) the diminution in the range of the horizontal force "storm time" variation at Pavlovsk as compared with Bombay; (*c*) as the intensity of the storm diminishes so does the rate of subsidence of the local-time variations from the first to the second day—in the case of the moderate storms at Pavlovsk the second day's variation is of about the same amplitude as that for the first day, but this is probably due partly to the local storm variation being different in type during the first few hours of the storm. On the whole, however, there is substantial proportionality between the general and local storm variations in storms of widely different intensities.

The other elements have been studied in like manner, but, as the results show no essentially new features, they are not reproduced here.

Part II.—*The Electric Current System and the Atmospheric Motions.*

§ 8. We have now summarised what are perhaps the main facts concerning the regular world-wide changes of magnetic force during a storm. The attempt will next be made to explain the more immediate cause of these magnetic variations. On the theory to be described, this cause is a system of electric currents which flow, in more or less horizontal strata, in the upper atmosphere. The external currents will, of course, be accompanied by corresponding induced currents within the earth, which will modify their effects. The

primary system, however, chiefly claims our attention at this stage. It may be resolved into two component systems, one of which, symmetrical about the axis, is responsible for the general storm variations, while the other is responsible for the local storm variations; for the present we ignore the more local currents which produce the irregular magnetic fluctuations.

(*a*) The lines of flow in the horizontal current sheet of the first system are approximately orthogonal to the magnetic meridians.* The current density is nearly uniform over a wide range of latitude north and south of the equator, though it gradually diminishes towards high latitudes. During the first brief phase of a storm (while the horizontal force is above the normal) the direction of flow is from west to east. Afterwards it is from east to west, and the intensity becomes much greater after the reversal.

Owing to the sphericity of the earth, and the decrease of current density with increasing latitude, the surface lines of magnetic force due to the current sheet, while running nearly horizontally along the magnetic meridians, will be more tightly packed near the equator than to the north and south, and lines will pass through the current sheet on each side of the equator. During the main portion of the storm, as a consequence, a numerical increase of the vertical force will be associated with the considerable decrease in the horizontal force. It is easily seen that such a symmetrical current system, in which the intensity varies with time approximately proportionately to the divergence of the horizontal force from the normal (*cf.* fig. 1), will account for the main features of the general storm variations.

(*b*) The current system to which the local storm variations (§ 3*b*, § 6) are attributed is represented, in crude diagrammatic form, by fig. 7. In this figure all complications arising from the inclination of the magnetic to the geographical axis are omitted, and also those due to the non-perpendicularity of these axes to the radius vector from the sun. The current system is depicted as viewed from the sun; it is regarded as stationary relative to the radius vector from the sun, whilst the earth revolves within the stationary current sheet. Each station thus experiences a diurnal magnetic variation depending on local time and, except in so far as our simplifying conventions are untrue, not otherwise dependent on longitude. In high latitudes, naturally, this approximates only roughly to the truth.

The local storm variation during the brief initial phase of a storm is different from that indicated in figs. 3–5, deduced from the consideration of whole (first and second) days of the storms. The occurrences during this early phase will be separately discussed in detail later (§ 1); in this paper attention will be confined to the main local storm variations existing

* The current-lines may also be compared with the isochasms (*cf.* footnote to § 4).

throughout the greater part of the storm, most intensely during the first day, and afterwards gradually subsiding. Moreover only the purely diurnal part of the local storm variations will be dealt with; the part which, in figs. 3 to 5, is merely local and accidental, or due to the partial suppression of the ordinary diurnal variations, will be neglected.

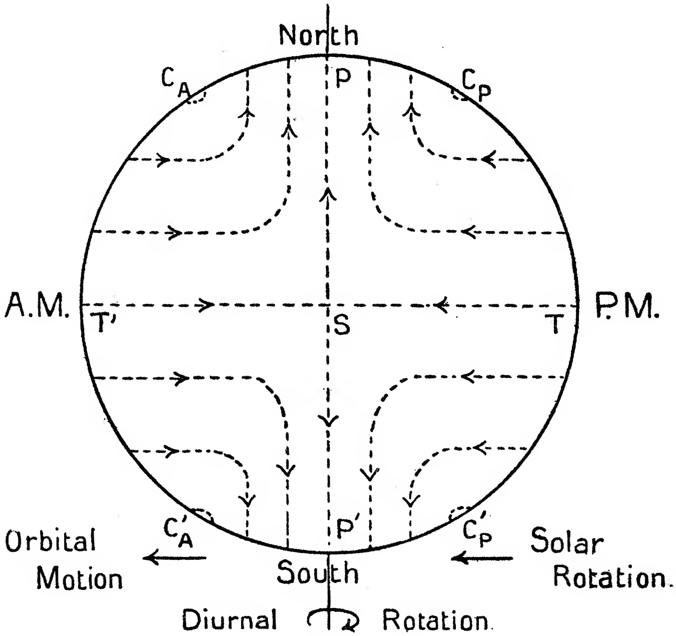

FIG. 7.

As represented in fig. 7, the local storm variation current system has a certain symmetry about the radius vector from the sun (or about a line not greatly inclined to this). The system is divided into four similar and self-contained quadrantal parts by the "solar meridian" and the equatorial planes; by the former is meant the meridian plane containing the sun, passing through the north and south poles P, P′, and through S, S′, the points at which the sun is in the zenith and nadir respectively. Over the hemisphere to the right of the solar meridian plane—as viewed from the sun—the local time is after noon; this hemisphere will hence be referred to as the P.M. hemisphere, and the opposite one will be correspondingly designated. The four quadrants may thus be distinguished as the north P.M., the north A.M., the south P.M., and the south A.M. quadrants. Again, the hemisphere on which the sun is directly shining will be termed the "day" hemisphere, and the meridian plane which divides this from the other ("night") hemisphere will be the "twilight meridian" plane, intersecting

the equator in T, T′. The meridian semicircles PSP′, PS′P′, PTP′, PT′P′ will be termed the midday, midnight, sunset, and sunrise meridians.

Along the equator the current is from east to west in the P.M., and from west to east in the A.M. hemisphere. The circuits, which bound the four quadrantal parts already mentioned, are completed in the solar meridian plane, along SPS′ and SP′S′. The remaining current lines in any quadrant are "simply conformable" with the bounding circuits, as is roughly indicated in the figure; they gradually reduce to points at C_P, C_A, C_P', C_A', which will be termed the "current foci." The position of these points perhaps depends somewhat on the intensity of the storm; in the case of the 40 storms dealt with in §§ 4, 5, they are approximately in latitude 55°–60°.

As regards the current density, this diminishes gradually to zero from T to C_P along the sunset meridian, and then increases again to the pole P; since the distance TC_P exceeds $C_P P$, the mean current density north of C_P must exceed that between C_P and the equator. Again, the current density at S and S′ is clearly zero, and steadily increases from these points towards the poles, P being the nearest point of the solar meridian circle to C_P.

Over the twilight meridian circle the surface magnetic force arising from the above external current system is along the meridian, southwards between C_P and C_P' on the sunset arc (*i.e.*, at 18 h. local time), and northwards between C_A and C_A' (at 6 h. local time); over the solar meridian circle (at 0 h. and 12 h.) the component of magnetic force along the meridian is zero. Between the current foci and the poles the force along the twilight meridian circle is opposite to that between the current foci themselves. Thus in the latitudes of the current foci the horizontal force variation suffers reversal, between and beyond these latitudes the variations are readily seen to be in general agreement with the observed variations depicted in fig. 3 (*b*, *c*).

As regards the disturbance vector in westerly force, or declination, this vanishes over the twilight meridian (6 h. and 18 h.), and is at its maximum over the solar meridian (0 h. and 12 h.),* being westerly and easterly respectively in the northern hemisphere at 12 h. and 0 h., and in the southern hemisphere at 0 h. and 12 h. There is no reversal of phase except at the equator, where the amplitude vanishes; the amplitude increases from this latitude towards the poles (*cf.* fig. 5).

Over the solar meridian circle (0 h. and 12 h.) the magnetic disturbance vector is horizontal, *i.e.*, the vertical force vanishes, as it does also everywhere along the equator. At the current foci the magnetic force is entirely

* Local time, being reckoned from midnight, is at 0 h. along the midnight meridian PS′P′, and at 12 h. along the midday meridian PSP′.

vertical, and at its maximum in amplitude. At C_P and C_A' it is downwards, while it is upwards at C_A and C_P'. It is readily verified that the corresponding diurnal variation in vertical force has its numerical maximum at 18 h. in both northern and southern hemispheres, though if considered radially the variation changes sign at the equator, as does the vertical component of the earth's general magnetic field. The amplitude vanishes at the equator and at the poles, and is at its maxima in the latitudes of the current foci. The curves in fig. 4 show general agreement with these and other conclusions to be drawn from fig. 7.

The internal current system associated with the atmospheric current sheet, illustrated in fig. 7, will reinforce the magnetic variation which the latter produces in the horizontal plane, and diminish the vertical force variations; at the same time the phase angles will be slightly modified. These, however, are matters which will be left for detailed consideration later (§ 1). The phases, or times of maximum, mentioned above, must in any case be regarded as only approximations.

§ 9. So far our concern has been either with facts drawn directly from observation, or, as in §8, the natural interpretation of these in terms of current systems. It is necessary now to proceed over slightly more hazardous ground, in the endeavour to explain the genesis of these current systems. The resolution of the magnetic variations and current sheets into two elementary systems, general and local, while convenient and legitimate, is, of course, somewhat artificial. Although, subsequently, a general view will be taken, in which the action will be considered as a whole, for the present the separation into two components will be maintained.

(*a*) The first current system (§ 8*a*), in which the circulation is round the parallels of latitude, is symmetrical about the earth's axis. The electromotive force (E.M.F.) impelling the current arises from inductive action occurring in the plane normal to the E.M.F., *i.e.*, in the meridian plane at each point. The most general action in this plane can be resolved into component parts, in one of which a vertical motion of the atmosphere takes place across the horizontal component of the earth's magnetic field, while in the other a horizontal current of air crosses the vertical magnetic force.

The atmospheric motions with which we are concerned take place at high levels (§ 12), of course, and are partly horizontal and partly vertical. At the moment, however, we are considering only that component of the whole system which is symmetrical about the earth's axis, and confined within the meridian planes. For the production of the current system of § 8*a* we must look mainly to vertical motions, which may be described somewhat as follows:—

During the fleeting initial phase of the storm, in which the increase of horizontal force occurs, the vertical movement is downwards. This gives rise to an easterly E.M.F., soon reversed, however, consequent upon a reversal of the atmospheric motion, which continues to be directed upwards for several hours. These motions may be more intense in high than in low latitudes, but the inequalities do not seem likely to be very great (§ 11). The vertical movement is effective only in conjunction with the horizontal component of the earth's magnetic field; an increase in the motion with increasing latitude may partly compensate for the corresponding diminution in the horizontal magnetic force, until a considerable latitude is reached (*cf.* figs. 1, 6). Variations of vertical motion with latitude may be accompanied by some horizontal flow along the meridians towards northern and southern latitudes of maximum upward motion. Such a flow would produce, in conjunction with the vertical component of the earth's magnetic field, electro-motive forces which in middle and low latitudes would slightly reinforce those due to the vertical movements. But the distribution of current is likely to be more influenced by variations of electric conductivity than of vertical motion.

The considerations which support these conclusions may be better appreciated after the discussion has been studied as a whole. It is desirable to indicate, however, why the alternative hypothesis is rejected, which would ascribe the production of the electromotive forces primarily to horizontal movements of the air. These movements would require to be directed away from the equator during the initial phase of a storm, and afterwards towards the equator. The reversal of motion must be assumed on either hypothesis, but an explanation of such a reversal seems less readily forthcoming in the case of horizontal than in that of vertical motions. It is to be expected, moreover, that such horizontal movements in one layer would involve opposite currents in neighbouring strata, in order to complete a circulation. The electromagnetic effects of these opposing motions would probably largely neutralise one another. The horizontal movements, also, would be least intense near the equator, where the vertical magnetic force is small. The resulting E.M.F. would thus be least where the observed E.M.F. is greatest; nor does a solution of these difficulties seem to be available from a consideration of the vertical motions which would be necessary to complete the circulation just referred to.

(*b*) [*Revised*, July 23, 1918.—The electric current system of the local storm variation (§ 8*b*, and fig. 7) must next receive attention. When the current lines are drawn for the combined system (§ 8*a*, *b*) it is readily seen that the second set of currents represents a crowding in of the current lines, running

nearly along circles of latitude, over the P.M. hemisphere, and a spreading out over the A.M. hemisphere. It is as though the latitude circles were distorted by having their centre or pole drawn somewhat towards the equator along the twilight semicircles, this being done symmetrically with respect to the equator, which remains a current line. Thus the local storm variation system may be regarded as indicating simply that the storm influences are more powerfully exercised over the A.M. than over the P.M. hemisphere. In accordance with the suggestions contained in § 9*a*, this might be attributed to a greater intensity of vertical motion over the former hemisphere. This course was the one originally adopted in this paper, and in addition a system of horizontal motions, such as might accompany the suggested unequal distribution of vertical motion, was outlined, by means of which the current system of fig. 7 might be explained. There are reasons, however, for expecting that no great inequalities of vertical motion may occur, and a simpler hypothesis is to hand, which attributes the inequality of current intensity between the two hemispheres to a greater electric conductivity over the P.M. than over the A.M. hemisphere. It may readily be shown mathematically that such a distribution of conductivity, in conjunction with uniform vertical motion, would produce a system of currents similar to that observed.]

Part III.—*The Origin of the Atmospheric Motions.*

§ 10. One further and yet more hazardous step must now be taken, in order to account for the atmospheric movements of § 9. The primary motion has been regarded as vertical—initially, for a brief interval, downwards, and afterwards, with greater intensity and for a longer period, upwards. It has also been seen (§ 7) that the more intense the storm, the more rapid is the progression of the characteristic "forced" phases of the general storm variation (as distinct from the phase of recovery). Considering these facts, and remembering the general connection between magnetic storms and auroræ, I am led to suggest the following explanation.

A magnetic storm is generated by the entry into the earth's atmosphere of numbers of electric particles, mainly or entirely of the same sign of charge. They penetrate to a more or less definite level in the upper atmosphere, this level depending on the density and composition of the atmosphere, and upon the physical nature and velocity of the particles. Their velocity, upon entry, is considerable, and the communication of their momentum to the absorbing layers imparts to these layers a downward motion, which is to be identified with the initial downward movement during magnetic storms (§ 9*a*). The ionisation of the layers by the impact of the electric particles may also

contribute to this downward motion; oxygen will be partly converted into ozone, with the double effect of diminishing the pressure and increasing the mean molecular weight of the layer.

The electric particles being mainly or entirely of one sign, the absorbing layer will become charged, and the mutual repulsion of the entangled ions will produce an expansion of the layer, *i.e.* an upward motion—just as in the case of an electrified soap bubble. The entry of particles into the atmosphere may continue for some hours, but, in general,* their downward momentum will be overborne by the upward expansion, which their absorption in the layer will only increase. Clearly, also, the more intense the injection, the more violent will be the initial depression and the subsequent expansion of the layer—the more rapidly will the charge be dissipated by its own mutual repulsion. The upward expansion of the layer is identified with that postulated by the theory of § 9, as occurring during the second phase of a magnetic storm. It will gradually cease as the charge becomes dissipated (part of the atmosphere itself being carried away in the process), and gravitational forces regain their control over the atmosphere.

Electric currents are generated in the atmosphere (and, secondarily, within the earth), while the movement of the air is producing or maintaining electromotive forces by induction; the particles which excite the motion also render the air conducting, by ionisation. The currents do not cease, however, as soon as the motion of the E.M.F. ceases—they will decay only gradually, owing to self-induction. Probably the earth currents decay more slowly than those in the upper atmosphere, and mainly govern the rate of recovery of horizontal force during the last phase of a storm. During the progress of the "forced" phases, self-induction, both within the atmosphere and within the earth (including mutual induction), will cause the reversals in the rate of change of the electric currents to lag behind the reversals of atmospheric motion.

§ 11. So far, the electro-mechanical effects have been described generally, without reference to locality. The distribution of intensity of the precipitation of particles into the atmosphere seems, however, not to be uniform over the earth. During a great storm, particles appear to enter the atmosphere practically all over its "surface," but a preference is shown for

* An unusually intense injection may temporarily reverse the upward motion, afterwards, however, increasing its rapidity. Perhaps the storm discussed by W. G. Adams ('Phil. Trans.,' A, p. 131 (1892)), of June 24, 1885, is an example of this, a marked increase of horizontal force occurring about 5½ hours after the commencement of the storm; this marked the beginning of what was practically a new storm as far as the "forced" phases were concerned. I hope to discuss this storm more in detail on a later occasion.

particular regions. When the intensity of the stream of corpuscles directed towards the earth is more moderate, as during the last phase (that of recovery) of a storm, these regions seem to receive almost all the particles which enter the atmosphere.

[The distribution of intensity of precipitation is, in a general way, identified, on the present theory, with that of the ionisation or electric conductivity, as described in § 9*b*. It is therefore more intense over the P.M. than over the A.M. hemisphere, and perhaps more intense and irregular in high latitudes than in low. Inequalities in precipitation will produce non-uniform distributions of ionisation and of surface density of charge, but the electric forces which the latter set up will, in the ionised, conducting layer, give rise to currents tending to restore a uniform surface density of electricity. The vertical motions resulting from this electric charge are therefore likely to be more uniform than the conductivity, inequalities in the distribution of the latter being more permanent.

If the distribution of the charge were perfectly uniform over the earth's surface, whatever its density, it would not disturb the electric potential gradient at the earth's surface. It is important to determine how nearly this condition is in fact approximated to, as the hypothesis of §§ 10, 11 must be rejected if it is found to involve appreciable modifications in the potential gradient during a magnetic storm. This question, which is one of some difficulty, is receiving careful consideration.

Besides the regular general inequality of precipitation, the magnetic variations suggest that there are considerable local and accidental inequalities, particularly in high latitudes. The marked variations in the distribution of conductivity thus produced are likely to have an important influence on the direction of the current lines, and to explain the remarkable magnetic variability in the polar regions of the earth. In high latitudes, localised precipitations continue for hours after the injection of particles has ceased over the major portion of the earth.—*Revised July* 23, 1918.]

§ 12. In many respects auroral phenomena seem to throw light on the above conclusions. Auroræ may themselves be the visible manifestation of vertical electric discharges* of unusual intensity. Precipitation would seem to be ordinarily confined to high latitudes, extending during a storm over a much wider area, but the injection may not be confined to the regions where auroræ are actually visible.

* Prof. Elihu Thomson has recently expressed this opinion in regard to auroral streamers and draperies, stating that when overhead they appear like rods viewed "end on": *cf.* 'Proc. Nat. Acad. Sci.,' vol. 3, p. 1 (1917).

In consequence of the connection with auroræ, it is natural to associate the atmospheric electric currents of § 8 with the auroral strata. According to the researches of Störmer, Vegard,* and Krogness,† the lower limit of height of the latter strata is about 90 kilom.; the layers of maximum auroral occurrence are between 100 and 110 kilom. above the earth's surface, and are not very deep. Auroral light is seen at times, however, at a height of 300 or even 400 kilom.

§§ 13–15. [These have been removed from the paper to bring it within the statutory limits of length for these 'Proceedings.' They dealt with the situation of the layers in which flow the currents responsible for the diurnal magnetic variations and for magnetic storms, and also with the solar ionising agencies which affect these layers. These subjects will now be dealt with in a separate paper, but it should here be explained that the ionising agents in the case of a magnetic storm, viz., the corpuscles referred to in § 10, were described as being emitted from the sun in streams; these were brought into relation with Mr. Maunder's important work‡ on the recurrence of magnetic storms. In consequence of the rotation of the sun these streams will approach the earth on the P.M. side, as indicated in fig. 7.—*July* 23, 1918.]

§ 16. The higher degree of magnetic disturbance over the P.M. hemisphere is not to be regarded as a mere consequence of the approach of the stream on that side of the earth, since the terrestrial magnetic field will modify the motion of the particles before they actually enter the atmosphere—many, indeed, will be deflected altogether away from the earth, according to Störmer's calculations. The latter have not yet been harmonised in detail with the observational facts regarding auroræ, nor with the indications of magnetic phenomena, as discussed in this paper. The analysis of the corpuscular paths is a very difficult mathematical task, however, and the success which Prof. Störmer has already obtained gives good hope that in time, as the initial postulates are brought into conformity with the physical evidence, so the mathematical theory will be perfected and will accord with observation.

* *Cf.* Störmer, 'Terrestrial Magnetism,' vol. 21, pp. 45, 153 (1916), and further references there given.

† *Cf.* Vegard and Krogness, 'Terrestrial Magnetism,' vol. 21, p. 169 (1916); in 'Jahrbuch der Radioaktivität and Elektronik,' vol. 14, Part 4, p. 384 (1917), Vegard gives a summary and extensive bibliography of auroral research.

‡ E. W. Maunder, 'M. N. R. Ast. Soc.,' vol. 65, pp. 22, 538, 666 (1904); vol. 76, p. 63 (1915), and other papers. The bearing of Dr. Chree's work on the recurrence-tendency shown also by quiet days was likewise discussed in the omitted sections; *cf.* C. Chree, 'Phil. Trans.,' A, vol. 212, p. 75 (1912); and vol. 213, p. 245 (1913).

The continuance of storms for several hours gives some idea as to the angular diameter of the streams of particles; one which takes five hours to traverse the earth must have a breadth, in the orbital plane, of about five million miles, or an angular breadth, from the sun, of about 3°. The sudden commencement of all the greatest storms seems to indicate that the most intense streams are somewhat sharply defined, at any rate on their forward side. They may be followed, in some cases (*cf.* that quoted in the footnote to § 10), by other and even more intense streams. More moderate storms often show no very marked commencement.

During the later stages of storms, as the intensity of the stream falls off, the corpuscles seem to be mainly drawn in towards a zone of high latitude on the P.M. side.

§ 17. One last point may be mentioned in connection with the local time diurnal variations in polar regions. The ordinary diurnal magnetic variations (§ 13) in high latitudes will be extremely small, on account both of the mechanical and ionisation factors (the latter depending on solar radiation). It is observed, indeed, that on the few days found to be really "quiet" in these latitudes the diurnal variations are extremely small. Normally the magnetic records are greatly disturbed, but these irregular fluctuations are found to be associated with a well-marked diurnal variation, the amplitude of which shows a marked relationship with the intensity of disturbance.* The type of the variations, moreover, is the same as that which is illustrated in figs. 3–5 (*b*), (*c*) (in particular, for Sitka), as derived from days of great magnetic storms. It would, therefore, seem that the local storm variation current system, which is of world-wide extent during great disturbances, is present over a more restricted region round the poles at nearly all times. The intensity and extent vary according to the intensity of disturbance, *i.e.*, according to the intensity of the moderate "stray" streamers, the particles of which are being drawn in towards the polar regions at the time. The current foci may approach towards or recede from the poles according to the strength of disturbance, but across the poles, between the current foci, the currents would seem to maintain their unidirectional character. It is this feature which explains the simple diurnal type of variation observed in polar regions. It should be added that the phase of Dr. Chree's Antarctic curves does not agree very closely with that which the present simple theory would suggest, though his vector diagrams are described in the expected direction; but the phase difference may well be due to the divergence, which we have so far ignored, between the magnetic and geographical axes (*cf.* § 1).

§ 18. In conclusion, I would add that it is an attempt to survey a some-

* *Cf.* Chree, Kelvin Lecture, 'Journ. Inst. Elect. Eng.,' vol. 54, p. 405 (1916).

what vague and complex subject, and to place certain definite features in theoretical relation to one another, that the discussion here presented should be judged. Some of the views expressed have been stated before in other contexts,* and in a more detailed and complete memoir I hope to refer to these and other theories on the subject. In this account I have mentioned, very briefly, only those papers which I have had actual occasion to use in the present discussion; but it is hardly necessary to state how much such an investigation must owe to the labours of others who have previously studied the many-sided phenomena dealt with.

It is a pleasant duty to acknowledge the assistance which has been placed at my disposal, in the execution of the computations necessary for this paper, by the Government Grant Committee of the Royal Society and by the Astronomer Royal.

The Diffraction of Electric Waves by the Earth.

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During the last 15 years, the problem of determining the effect at a distant point of the earth's surface due to a Hertzian oscillator emitting waves of a definite frequency has been the subject of numerous theoretical investigations.

When certain assumptions of a physical character have been made, the problem is of a definitely mathematical type; it is in fact reduced to the problem of finding an approximate formula for the sum of a certain complicated series of an oscillatory nature; we shall summarise the principal methods which have been devised for dealing with this series.

The method of Poincaré† and Nicholson‡ is to replace the series by an integral and then to obtain an approximate value for the integral by means of the calculus of residues. The analysis employed by them, though

* Since writing this paper, for example, I have noticed that the symmetry of the disturbance variation about the solar meridian plane (§ 6) had been remarked by van Bemmelen in 1903 ('Terrestrial Magnetism,' vol. 8, p. 153), who also (*ibid.*, vol. 5, p. 123) refers to a theory of "current-vortices" (*cf.* § 11) by Schmidt ('Met. Zeitschrift,' 1889, p. 385).

† 'Palermo Rendiconti,' vol. 29, pp. 169–260 (1910).

‡ 'Phil. Mag.,' 1910, *passim.*